Benchmark: dynamics of a flexible five-bar mechanism

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Abstract

This document presents a flexible five-bar mechanism as a benchmark problem for the dynamic analysis of flexible multibody systems.

1 Description of the five-bar mechanism

The five-bar mechanism considered as benchmark problem is a modification of the 5-bar linkage originally proposed in [Pagalday, 1994]. The flexible mechanism is composed of 4 rigid bars (one of them fixed) and a flexible bar (marked in green in figure 1).

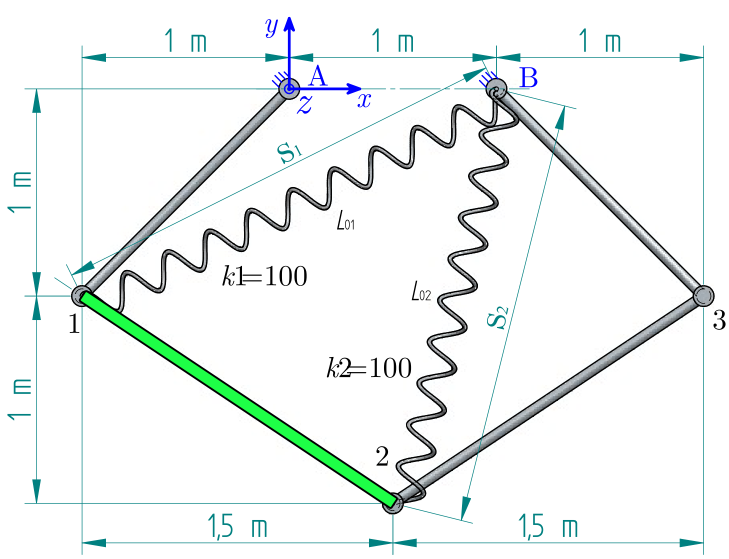

Figure 1: The flexible five-bar mechanism

The five bars are constrained by five revolute joints located in points A (origin of global coordinates), 1, 2, 3 and B, with A and B attached to the ground. The masses of the rigid bars are $m_{A1} = 1$ kg, $m_{23} = 1.5$ kg, $m_{3B} = 1$ kg and the polar moments of inertia are calculated under the assumption of an uniform distribution of mass. The mechanism is subjected to the action of gravity, $\mathbf{g} = \begin{bmatrix} 0 & -9.81 & 0 \end{bmatrix}^{\mathrm{T}}$ m/s^2, and two elastic forces coming from the springs. The stiffness coefficients of the springs are $k_1 = k_2 = 100$ N/m and their natural lengths are chosen $L_{s1} = \sqrt{2^2 + 1^2} = \sqrt{5}$ m and $L_{s2} = \sqrt{2^2 + 0.5^2} = \sqrt{17}/2 = \sqrt{4.25}$ m, coincident with the initial configuration shown in Fig.1.

The flexible bar is a prism made of steel ($E = 2.0694 \times 10^{11}$ Pa, $\nu = 0.288$ and $\rho = 7.829 \times 10^3$ kg/m^3) with a length of $L_{12} = 0.5\sqrt{13}$ m and a rectangular cross-section of 7.29×10^{-3} m by 14.58×10^{-3} m, with its largest cross-sectional dimension aligned with the axis of the revolute joints. The points which define the revolute joints in the flexible bar are placed at the center of each tip section. The revolute joint vectors are defined between the center of the section and the middle point of the edge according to figure 2. The faces of each of the tips are considered rigid, i.e., the nodes contained in them behave as a rigid body.

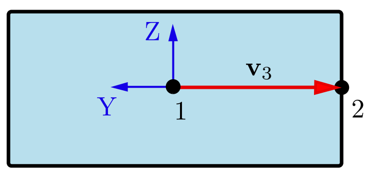

Figure 2: Revolute joint definition in tip face of the flexible bar

Rayleigh damping is considered in the flexible bar, with $\alpha = 0$ and $\beta = 5 \times 10^{-5}$ as mass and stiffness coefficients, respectively.

The reference solution is calculated by means of a FFR ALI3-P formulation, where deformation modes are calculated from a modal analysis with the Y translation and the X and Z rotations blocked for the tip faces.

2 Five-bar mechanism: dynamic problem description and solution

The dynamic maneuver consists in a 5-second simulation of the mechanism subjected to the action of gravity and the spring forces, being the initial velocities null and the flexible bar initially undeformed. The response of the system is monitored by means of the following magnitudes calculated at position, velocity and acceleration levels:

- Bar 12 midpoint deflection $\mathbf{d}_{12}$. It is measured as the difference between the global position of a material point placed at the undeformed center of mass of the flexible bar (see point 5 in figure 3) minus the position of the middle point of the segment connecting points 2 and 1:

$$\mathbf{d}_{12} = \mathbf{r}_5 - \frac{\mathbf{r}_2 + \mathbf{r}_1}{2} \tag{1}$$

Files `DynDeflecPos.csv`, `DynDeflecVel.csv` and `DynDeflecAcc.csv` contain the evolution of the midpoint deflection over time for x and y coordinates at position, velocity and acceleration levels. Additionally, this information is displayed in figure 4.

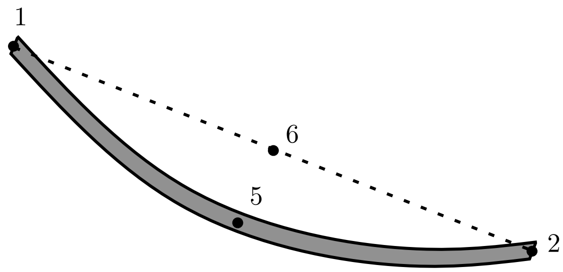

Figure 3: Deflection measurement

- Coordinates of point 2. The position, velocity and acceleration of point 2 are stored in `Dynr2Pos.csv`, `Dynr2Vel.csv` and `Dynr2Acc.csv`, and displayed in figure 5.

The reference solution provided for the dynamic analysis has been obtained with a time step of 5×10^{-5} s with two different floating frame of reference formulations (ALI3-P and Matrix R), with the flexibility of bar 12 parameterized by means of 50 modes of deformation coming from a modal analysis of a mesh with a maximum element size of 2×10^{-3} m. In the modal analysis, the translation along Y and rotations around X and Z at the tip faces are blocked.

3 Benchmark error calculation

Six measurements of error are proposed for this mechanism, each one measuring the integral over time of the deviation of a measurement with respect to its corresponding reference. Three errors measure deflection deviations:

$$error^1 = \int_{t_0}^{t_f} \left\| \mathbf{d}_{12} - (\mathbf{d}_{12})_{Ref} \right\|_2 \mathrm{d}t \tag{2}$$

$$error^2 = \int_{t_0}^{t_f} \left\| \dot{\mathbf{d}}_{12} - \left(\dot{\mathbf{d}}_{12}\right)_{Ref} \right\|_2 \mathrm{d}t \tag{3}$$

$$error^3 = \int_{t_0}^{t_f} \left\| \ddot{\mathbf{d}}_{12} - \left(\ddot{\mathbf{d}}_{12}\right)_{Ref} \right\|_2 \mathrm{d}t \tag{4}$$

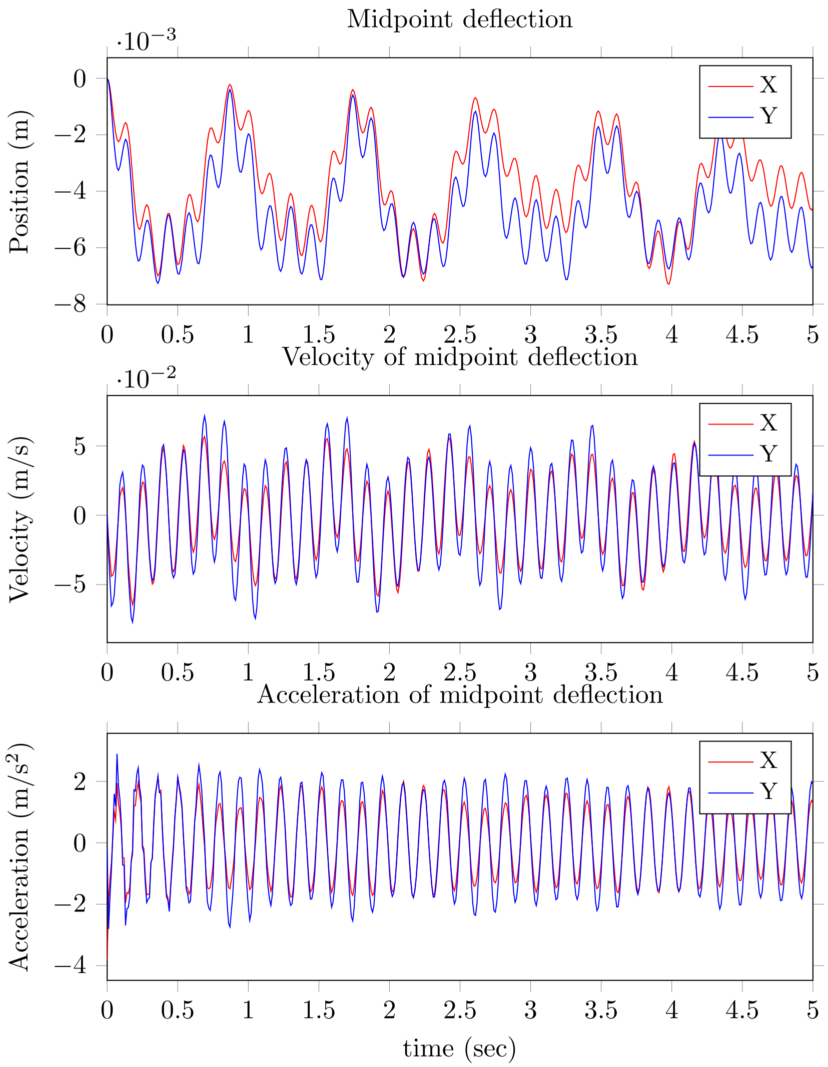

Figure 4: Dynamic response: position (top), velocity (middle) and acceleration (bottom) of the midpoint deflection.

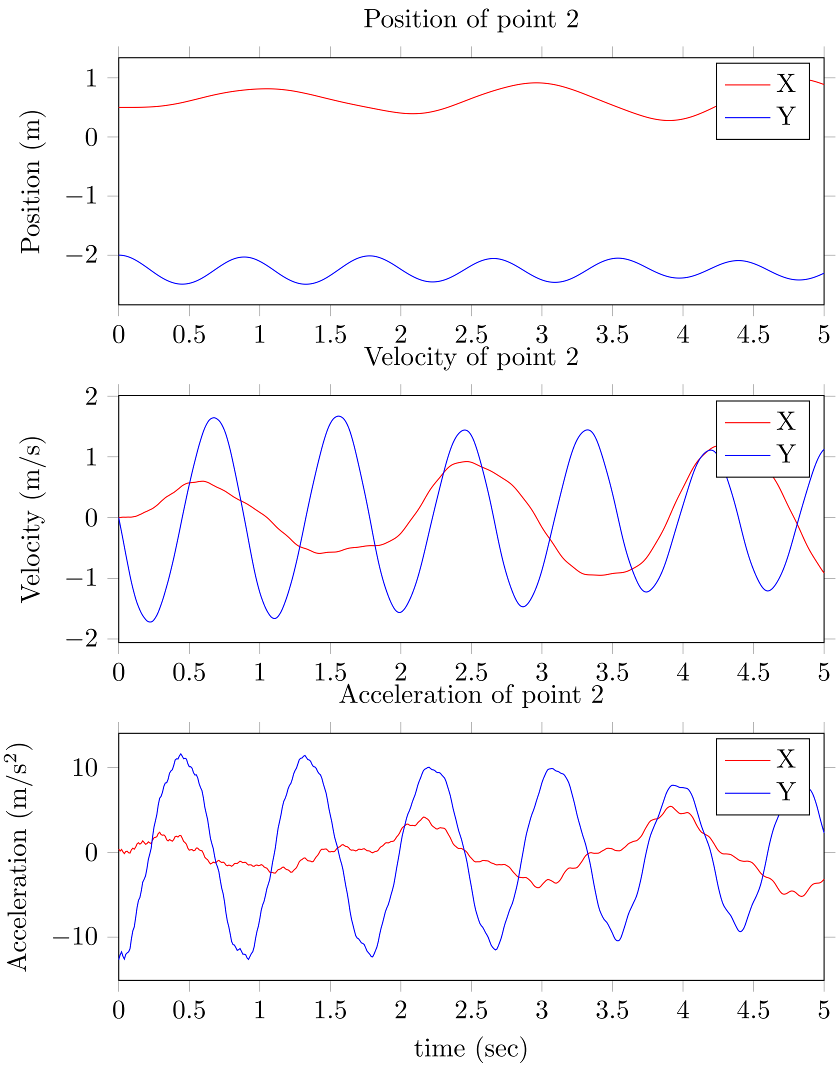

Figure 5: Dynamic response of point 2: position (top), velocity (middle) and acceleration (bottom).

and another three measure deviations in $\mathbf{r}_2$:

$$error^4 = \int_{t_0}^{t_f} \left\| \mathbf{r}_2 - (\mathbf{r}_2)_{Ref} \right\|_2 \mathrm{d}t \tag{5}$$

$$error^5 = \int_{t_0}^{t_f} \left\| \dot{\mathbf{r}}_2 - (\dot{\mathbf{r}}_2)_{Ref} \right\|_2 \mathrm{d}t \tag{6}$$

$$error^6 = \int_{t_0}^{t_f} \left\| \ddot{\mathbf{r}}_2 - (\ddot{\mathbf{r}}_2)_{Ref} \right\|_2 \mathrm{d}t \tag{7}$$

with $\|\cdot\|_2$ denoting 2-norm.

Valid solutions should be below the maximum error tolerance for each error measurement. The maximum tolerances for each error are displayed in table 1.

$$error^i \leq \epsilon_i, \quad i = 1, ..., 6. \tag{8}$$

A single condition is required for publishing in the benchmark platform. For this purpose, the following single condition can be followed from conditions (8):

$$error = \sum_{i=1}^{6} \frac{\epsilon_6}{\epsilon_i} error^i \leq 6\epsilon_6 = 6 \tag{9}$$

	Maximum error tolerance (ϵ_i)
$error^1$	2×10^{-3}
$error^2$	6×10^{-3}
$error^3$	1
$error^4$	2×10^{-3}
$error^5$	5×10^{-3}
$error^6$	1

Table 1: Maximum allowed error

The file `results.txt`, attached to the solution, provides the errors $error^i$ in the first six columns (with $i = 1, ..., 6$) and the total error (9) in column 7.

References

[Pagalday, 1994] Pagalday, J. (1994). *Optimizacion del comportamiento dinamico de mecanismos*. PhD thesis, Escuela Superior de Ingenieros Industriales de S. Sebastian.